

# The concept and Evolution of Infinity

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**Introduction:** The concept and evolution of infinity has been an important aspect not only in mathematics, but also in all scientific subjects leading to many marvelous inventions and discoveries. Like Zero and Numerals, what the world use today, infinity, has also been invented by the Indians, when such subject never had never been conceived, discussed and recorded in the west. The concept and evolution of Zero and Infinity have been interconnected, philosophical, ontogenic and finally mathematical. Therefore, it is pertinent to examine as to how they could have been invented such concept. An attempt made in this paper to study about it, in the historical perspective.

**Infinity:** Unlike the other ancient civilizations (of which the so called Babylonian, Assyrian, Egyptian were already extinct), the Indian civilization is continuing one and has been enriched Sanskrit literature, which is information packed in all aspects. Though, a section of western and certain group of school to dub it as warehouse of myth, mythology, fables and so on, a careful study has revealed many interesting and informative results. Particularly, Vedas and Puranas have solved many mathematical riddles about the Number system, Zero and Infinity. Man has always a fascination for the nature, world, mountains, rivers, flora, times fauna and their interaction with man. Many times, he has tried to imagine that is unimaginable; comprehend that incomprehensible and represent symbolically to express and convey its meaning to others.

The ancient Indian mind had been imbibed with such ideas, which were not only philosophical, but also scientific. The thinking processes were always about the origin and end. Every Upanishad or Purana explains succinctly explains this aspect cosmologically with the description of creation (प्रभव - Prabhava / निर्मानम् - Nirmanam) and destruction (प्रलय - pralayaya / निर्मूल्यम् - Nirmulyam). Such acts occur cyclically and continuously. Therefore, in the analysis of numbers and their operations, they must have invented the conceptual entities of zero and Infinity as the beginning and end of the numeric system.

**Who used Infinity First?:** The western sources as usual point to the Greeks, particularly, Anaximander (c.611-545 BCE), Leucippus (400 BCE?) and Zeno of Elea (c.450 BCE) for the conception Infinity. But, their conception was superficial and they had not specifically mentioned anything about it. As they were tired of thinking about Infinity, "...**the classical mathematicians banned from rigorous thought, the infinite in all its forms - infinitesimal, the infinitely great, and infinite multiplicity**"(1). They, "**had avoided it, by their elegant method of exhaustion**".

Even after the introduction of Calculus in 17<sup>th</sup> century, the western mind was puzzled with Infinity and Infinitesimals. The puzzle persisted for nearly two centuries, until Cauchy (1789-1857) and Weierstrass (1815-97) showed how the awkward notion could be eliminated" (2). But. in the case of Indians, there was no such mental setup, as they could think of very big and small numbers.

The Indians have definitely conceived the idea of Infinity equated BCE) with that of Brahman and Atman. There are many Vedic (3500 BCE) and Upanishadic (2000 BCE) references to prove this fact. Mathematically, Brahmagupta (628 CE) was the first who attempted division by zero and named such resultant quantity "khacheda" (destroyer of zero).

The first usage of Infinity in mathematical operations is found in the Bija Ganita of Bhaskaracharya (1114- 1185 CE). He mentioned it as khahar (remover of zero). He mentions:

"In this quantity consisting of that which has cipher for its divisor, there is no alteration, though many may be inserted or extracted; as no change takes place in the infinite and immutable God, at the period of the destruction or creation of worlds, though numerous orders of beings absorbed or put forth" (3),:

implying that he knew that  $n/0 = \infty$ , and  $\infty + n = \infty$ .

Ganesa (1545 CE) notes that  $n / 0$  is:

".....an indefinite and unlimited or infinite quantity; since it cannot be determined how great it is. It is unaltered by the addition or subtraction of finite quantities; since in the preliminary operation of reducing both fractional expressions to common denominator, preparatory to taking their sum or difference, both numerator and denominator of the finite quantity vanish" (4).

Krishna has also recorded:

"As much as the divisor is diminished, so much is reduced the quotient increased. If the divisor is reduced to the utmost, the quotient is to the utmost increased. But, if it can be specified, that the amount of the quotient so much, it has not been raised to the utmost: for a quantity greater than that can be assigned. The quotient therefore, is indefinitely great, and is rightly termed infinite" (5).

Thus, it is evident that they have clearly recognized and used Infinity in the mathematical operations leading to indeterminate forms. Limits. Series etc.

**Words Used for Infinity and their Significance:** The words used for Infinity are Aanantyam. Anantata. Apartā, Simabhav, Asimta and Parardhye (6).

Anantya आनन्त्य	Endlessness; infinity
anatya अनन्त्य	Endless, eternal, infinite.
Aanantyam अनन्त्यम् आनन्त्यम्	1. Infinity, endless (in time, space or number) as in "anantyadh vyabhi characha" (Kavya prakasa). 2. Boundless 3. Immortality, eternity
Anantata अनन्ता	1. the earth (the endless) 2. The number One 3. Noun of Parvati, 4. noun for various plants like Sakhi, Aantamulam, Dhurva etc.
Anantam अनन्तम्	1. The sky, atmosphere 2. Infinity 3. Absolution 4. The Supreme Spirit, Brahma (Parabrahmam)
Aparta अपवारता	1. Shoreless 2. Boundless, unlimited 3. Inexhaustible. immense 4. out of reach 5. Difficult to be crossed, difficult to be surmounted or overcome
Simabhav	The idea. nature or appreciation of limits (i.e, the highest or utmost limit)
Asimata असीम असीमित असीमम्	endless, boundless limitless, Infinity

Monier Williams adds the following meanings to the word ananta - 23<sup>rd</sup> lunar asterism and a periodic decimal fraction (7), implying its connection with mathematics.

Thus, the words expressed denote specifically, the "Infinity" that cannot be incomprehensible, but understood with already conceptualized and known typical examples.

**Is Infinity a Number?:** The Hindu mathematicians initially considered Infinity as a number like Zero, though modern mathematicians do not consider so. As the quantity of "limitless boundless very Great" is limited mathematically. Infinity" is finitely defined with the conceptual development, we can understand that Infinity can only be approached,

but not reached! The operations of Infinity with zero and other numbers explained by Bhaskara II, Ganesa and Krishna clearly show that slowly, they too considered it as a mathematical entity, but not a number. Western mathematicians always asserted that infinity is not a number. Any number how much it might be great is Finite only. Georg Cantor in his set theory shows the existence of many infinities. He also discovered the **transinfinite numbers**, which are related to the concept of infinity. This again amounts to acceptance of many distinct infinities, but they are identified finitely.

**Operations with Infinity:** The following operations of infinity with Zero, any number and Infinity itself are considered to understand its uniqueness in mathematical processes and logic.

$\infty + \infty = \infty$ $\infty - \infty = \infty$ $\infty \times \infty = \infty$ $\infty \div \infty = \infty$	The operations of Infinity with Infinity prove that such operations do not change anything, as the result is Infinity only.
$n + \infty = \infty$ $n - \infty = \infty$ $n \times \infty = \infty$ $n \div \infty = \infty$	The same concept is extended to the operation of any number with Infinity
$\infty + 0 = \infty$ $\infty - 0 = \infty$ $\infty \times 0 = 0?$ $\infty \div 0 = \infty$	The operations of Infinity with Zero is interesting. Here, the last operation makes us to consider that Infinity may also be a number like Zero.

Some interesting questions about infinity: any operation of infinity with infinity results in infinity only.

$$\begin{aligned}\infty + \infty &= \infty \\ \infty - \infty &= \infty \\ \infty \times \infty &= \infty \\ \infty \div \infty &= \infty\end{aligned}$$

This is exactly defined in “purnasya purnamathaya purnamevavashiyate” (If the whole is removed from the whole, only the whole will remain).

1.  $n^n = n$ ;  $n \times n \dots$  is not  $= n$ .  
but,  $\infty \times \infty = \infty$ ;  $\infty \times \infty \times \infty = \infty$ ;.....and so on.
2. Again,  $n + n = 2n$ ,  
But  $\infty + \infty = 2\infty$ ,  $\infty$  only.
3. Similarly,  $n \div n = 1$   
But,  $\infty \div \infty = \infty$  or indeterminate.  
Thus, we can note the operations with infinity is a clear departure from the law of finite numbers.

4. We know  $a \div 0 = \infty$  and  $b \div 0 = \infty$

But,  $a \div 0 \neq b \div 0$

Similarly,  $1 \div 0 = \infty$ ;  $2 \div 0 = \infty$ ;  $3 \div 0 = \infty$ ; etc., but  $\infty$  in each case is different..

Its nature is best explained as follows:

We know  $1 \times 0 = 0$

$2 \times 0 = 0$

$1 \times 0 = 2 \times 0$

It can be written as  $1 / 2 = 0 / 0$

$2 / 1 = 0 / 0$

Therefore,  $1 / 2 = 2 / 1$ , thus, cross multiplying,  $1 = 4$

In the same way, we can show,  $4 = 9$ ,  $9 = 16$ ,  $16 = 25$ ,  $25 = 36$ ,  $36 = 49$ ,  $49 = 64$  and so on. Incidentally, it may be noted that all these numbers are squares of 1, 2, 3, 4, 5, 6 and so on.

This is because the zero obtained from  $1 \times 0$  and  $2 \times 0$  are not equal, as imagined. Similarly  $0/0$  is infinitely great as zero is divided by zero.

5. Similarly, infinities are also different, as has been noted above. Therefore, let us denote separately:

$0 \times \infty' = \infty$	$1 / 0 = \infty$ , but it is different $\infty'$
$0 \times \infty'' = \infty$	$2 / 0 = \infty$ , but it is also different $\infty''$

.....and so on.

Thus,

$\frac{1/0}{2/0} = \frac{\infty'}{\infty''}$ or	$\frac{0}{0} = \frac{\infty'}{\infty''}$ or
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Or  $0 \times \infty' = \infty''$

i.e.,  $1 = 2$ .

These fallacies occur, because, the infinite in each case is different, but it has unique quality or absorbing every number in it to be infinity always. This is best explained by the following illustration:

Let  $1 \times 0 = 0$   
 $0 \times 1 = 0$   
 Therefore,  $1 \times 0 = 0 \times 1$   
 $\frac{1}{0} = \frac{0}{1}$   
 Hence,  $1 = 0$

This is absurd, because.  $1 \times 0 \neq 1$ , as infinity in each case is different. Therefore, the first two equations are incorrect, though mathematically appear to be correct.

**Numbers without infinity:** what would happen, if mathematicians, physicists and scientists do not have infinity for their operations? Can they imagine such intricate concepts, evolve hypotheses and formulate theories of Relativity, Length contraction etc., in physics, Infinite series, Infinite vector, Infinite space, Axiom of infinity, set theory, Continuum hypothesis in mathematics. No, they are very much essential for the mathematical and logic processes. Infinities and infinitesimals are much required for scientific notations and derivations. As numbers cannot exist without zero, they cannot be operated effectively in higher mathematics and physics without infinity. Definitely, "Set theory" is an accommodative method of using infinities effectively in mathematics and "Reformalization" in physics.

**The Relation between Zero and Infinity:** Mathematically. Zero and Infinity have very close relation, though such relation is difficult to express mathematically for layman to understand. Narayana C.1356 CE in his works mentions about the operations with zero. In English, Greek and western linguistic connotation, zero means nothing, naught, nil, nullum, nonentity, because. from Arabic al-sifr = cafra = to be empty, but in Indian connotation it is "purnam" i.e. complete. In other words, from which all numbers emanate and end with infinity. No where in this world, such concept exists in any literature, except in Sanskrit literature.

Again, Indians were first to conceive the concept of Infinity and represent it mathematical operations. While Brahmagupta (628 CE) named it as kha-cheda (the quantity with zero denominator), Bhaskara II (c.1150 CE) called it kha-hara (zero as divisor).

"Statement: Dividend 3. Divisor 0. Quotient the fraction  $3/0$ . This fraction of which the denominator is cipher, termed an infinite quantity.....In this quantity consisting of that which has cipher for its divisor, there is no alteration, though many may be inserted or extracted: as no change takes place in the infinite and immutable God, at the period of the destruction or Creation of worlds, though numerous orders of beings are absorbed or put forth" (7).

Thus, mathematically, according to him,  $\infty / 0 = \infty$  and  $\infty + \infty = \infty$ .

Ganesa (1545) and Krishna have also noted the operations of infinity. Both prove  $a/0 \pm K a/0$ . Krishna also shows that  $a/0 = b/0$ . These clarify many operations with Infinity dealt with elsewhere in this paper.

To differentiate the different types of Infinities Krishna explains,

".....Thus, if the radius be 120: and the gnomon be 1, 2, 3 or 4; the expression deduced from the proportion, as sine of sun's altitude is to sine of of zenith distance, so is gnomon to shadow becomes 120/0, 240/0, 360/0 or 480/0. Or the gnomon be, as it is usually framed, 12 fingers, and radius be taken as 3438, 120, 100 or 90. the expression will be 41256/0, 1440/0, 1200/0 1080/0, which are alike infinite" (7).

So he says though Infinities result from the operation of dividing different numbers by 2800 are not equal, but they are not equal, but, they are alike infinite having specific value. Therefore, in mathematical operations, the Infinities arising during different operations are not equal, but they have different values and different. This concept is very Important in understanding and solving many puzzling problems discussed above.

Therefore, the operations of Infinity are succinctly explained. These operations are arrived at by the Hindu mathematicians lucidly with the philosophical basis supported by the Vedic texts.

Indians had clear distinction between the different usage of the concept of zero. Even in Patiganita (arithmetic), the operation of division by zero is not recognized as valid. but treated in Bijaganita (algebra). Narayana (1356) clearly points out the fact:

"Here in patiganita, division by zero is not recognized and therefore, it is not mentioned here. As it is of use in bijaganita, I have mentioned division by zero in my Bijaganita" (Ganita kaumudi).

Whereas, the western mathematicians do not have such differentiation. That is why they have noted the so called fallacies in some algebraic problems, because, there, they have unwittingly at tempted to multiply both sides of a equation by zero and equate the equals (8), thus leading to absurd results of  $1 = 0$ ,  $1 = 2$  and so on!

**The Form of Infinity**: Infinity is written two Zeroes (characteristically small zero touching a big zero each other) joined together horizontally. It is claimed that John Wallis (1616-1703) was the first to introduce the symbol  $\infty$  for Infinity in his works "Arithmetica infinitorum" and "De Sectionibus conicis" (9). It is claimed that he adopted that symbol taken from a Roman inscription dated to 36 C.E, where it was used to denote 10,000. It was inscribed as two zeros joined together placed within a frame (10). Initially, there was opposition to his usage the symbol used had no consistency with the concept and understanding of infinity. Voltaire criticized that, "**....the use of the love-knot does not add to our understanding of the concept**". However, Newton (1642 - 1727) and Gottfried Wilhelm Vo Leibniz encouraged its use by the mathematicians. But, about the infinity, the mathematicians were very much confused (11).

However, in the Indian context, the Indian mathematicians had clear idea about infinity, as the Indians had already vividly defined, explained and symbolized. It is pertinent to

mention that the name of the First Chief Serpent Adi Sesha with thousand hoods is Ananta and its coiled form is nothing but the Infinity Symbol. It is associated, compared and equalled with Krishna (Bhagavat Gita X.28-29), Vishnu (forming couch in the Ocean of Milk), Churning of Ocean leading to the Creation, Balarama, Varaha - the third incarnation of Vishnu at the beginning of kalpa. Such Coiled Snake form is depicted very often in sculptures and erected on the banks of rivers, tanks and oceans; near and inside temples and even in the inscriptions. Thus, the symbol of infinity obtained from Ananta or Sesha is not unimaginable incomprehensible.

Coming to Wallis'  $\infty$ , it should be analyzed how all of sudden he could have used it to denote much avoided infinity (in 1655, 1657 or 1665). Karl Menninger points out that the Roman numeral for 100 million consisted of the symbol  $\infty$  placed within frame, which itself originated from an extension of the numeral for 10.000. It well known that Romans did not have any zero to inscribe in their inscriptions. Therefore, it must have sheer accident to justify for his choice of  $\infty$  from it. In fact, he was a researcher in religion and knew about Hindu mythology. The familiarity of Hindu scriptures to the western scientists and mathematicians is well known. Oldenberg, the famous Sanskrit scholar was corresponding with Newton and others. Wallis was corresponding with Newton and other contemporaries (12), who were very much familiar with Hindu Mythology, Mathematicians, particularly, the Chronology, it is not surprising that he should have used such symbol to denote Infinity. Newton knew Bentley (13), who denounced Indian Astronomy Joseph Priestley, another scientist was also writing against Indians (14). Therefore, their anti-Hindu bias is intriguing.

**The Opposition to Infinity by the West:** The Infinity had been opposed by the west from the Greeks to modern scientists for the best reasons known to them. The writers tactfully record that while the Greeks avoided Infinity by their "elegant method of exhaustion", the modern mathematicians with saying that it was an "illusion". D'Alembert (1717-1783) looked upon infinity as nothing but a limit, which the finite approaches without ever reaching it. When Lagrange (1736-1813) endeavoured to free the calculus of its metaphysical difficulties, by resorting to common algebra, he avoided the whirlpool of Charybdis only to suffer wreck against the rocks of Scylla. The algebra of his day, as handed down to him by L. Euler, was founded on a false view of infinity (15).

Besides, some interesting views of mathematicians are quoted to bring out their mental set up:

The notions of Infinity or actual infinite was subjected to radical changes during 19<sup>th</sup> century. As late as in 1831, K.F. Gauss expressed himself, thus:

**"I Protest against the use of infinite magnitude as something completed, which in mathematics is never permissible.** Infinity is merely a facon de

parler, the real meaning being a limit which ratios approach indefinitely near, while others are permitted to increase without restriction" (16).

Gauss' contemporary, A.L Cauchy, likewise rejected the actual infinite, being influenced by the 18<sup>th</sup> century philosopher of Turin, Father Gerdil (17).

In 1886 Gerg Cantor occupied a diametrically opposite position, when he said:

"In spite of the essential difference between **the conceptions of the potential and actual infinite**, the former signifying a variable finite magnitude increasing beyond all finite limits, while the latter is a fixed, constant quality lying beyond all finite magnitudes, it happens only too often that the one is mistaken for the other..... **Owing to a justifiable aversion to such illegitimate actual infinities** and the influence of the modern epicuric-materialistic tendency, **a certain horror infiniti has grown up in extended scientific circles**, which find its classic expression and support in Gauss, yet it seems to me that the consequent **uncritical rejection of the legitimate actual infinite is no lesser violation of the nature of things**, which must be taken as they are" (18).

**Why they had difficulty in thinking Infinity or beyond Infinity?:** Infinity offers limitless, boundariless and innumerable options for thinking processes to conceive new ideas, evolve new hypotheses and formulate new theories. But, avoidance would only lead to the definiteness, i.e, limitation, restriction and control of the thought processes. Scientists know that the so called Greek Scientists like Pythagoras and western ones Kepler, Newton, De Broglie, Einstein, Kramer have committed several blunders in their inventions and discoveries, but covered up by the apologetic self explanations and philosophical musings masqueraded under the scientific garb, as they were dealing with intrinsic phenomena.

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Thinking Infinity, feeling Infinity and even reaching or attaining Infinity makes man free - Freedom with all rights. He need not care for anything. Mark, however, he thinks infinitely the Finite, finally, he to symbolizes such conceptions about Infinity in material forms. It makes man to think infinite worlds, infinite Solar systems, infinite Gods and so on. Such thinking may question the religious dogmas of the westerners (Jews, Christians and Muslims), but not Indians. Therefore, there is nothing strange in their opposition to the idea of Infinity.. As they opposed Zero, they opposed Infinity also, but scientists have ultimately understood their importance and readily started incorporating. Thus, Infinity is also being respected by them. However, they want to have not one Infinity, but several, why infinite Infinities. Very good in deed!

And now, one can note how all these ideas have been as consistent as ever before with the Cosmology and mathematical calculations of the Indians.

**The Basis of Such Operations:** Who must have thought of Infinity its operations, and also thought about very very small and very very great numbers only, could have conceived it. The ancient Indians have on record to prove that they knew numbers up to

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- ❁  $10^{12}$  - Parartha (Yajur Veda,
- ❁  $10^{22}$  - Taitreya Upanishad,
- ❁  $10^{53}$  - Tallaksana (Lalita Vistatara),
- ❁  $10^{67}$  Valmiki Ramayan,
- ❁  $10^{140}$  = asankhyaye (Kacchyana's Pali Grammar).

They are evidently multiples of 10. When a given number is divided successively by smaller numbers, the resultant is nearly equal to Infinity or approaches Infinity. This is amply proved in their calculation of time up to Brahmanda (a day of Brahma), the age of earth, the years of four yugas etc.

Very often God is considered as Infinity and every human being, a Zero (here simply nothing) is attempted to approach Him. God also compared with 1, as without which the zeroes (human beings) are valueless. Thus, the words anantata and anantam denote both One and Infinity. As God is everything, all are considered to have been originated from and ended with Him. Similarly, all originate from Purnam / Sunyam (Zero = Hiranya garba / Golden Egg) and end with anantata / anantyam / anantham (Infinity). As man can only approach God, but not reach Him or become God Himself, no number, whatever bigger it may be, reach Infinity or become Infinity, but only approach infinity. **That is why figuratively, Vedas say that one should have ananta dhrusti (Unlimited Vision) in ananta jyoti (Infinite Light) to approach anantata (Infinity) to get ananta-anandh (Infinite Bliss).**

In Chandayoga Upanishad (19), the conversation between Satyakama Jabala and his guru reveals many interesting details about Infinity. Though, it appears to be philosophical, the mathematical mind can very easily understand the significance of it. The Fire instructs Jabala that one who meditates on endless (ananatavan) becomes endless (anantaván). One who knows that this one foot of Brahman consisting of four parts, thus and meditates on it as the Endless (anantavata), wins the endless regions (4.6.4). That is whoever associates with Infinity becomes Infinity. Thus, any number is operated with Infinity, the number loses its identity and the result is infinity. Thus, the above operations are clearly proved.

Again the Fires (the Garhapatya, Anvarhayapacana and Ahavaniya) tell him that **Prana (life) is Brahman, क - ka (Joy) is Brahman, ख - Kha (ether / Infinity) is Brahman** (4.10.4). Satyakama Jabala, characteristically replies that he understands that Prana is Brahman, does not understand that Ka and Kha, the Fires clarify that Prana (life) and the Akasa within the heart are related to it (4.10.5). That is the pleasure centered in Akasa, not the ordinary pleasure and the Aksha the centre of happiness, not the

physical Akasa, are to be meditated upon. **Kha qualifying Ka limits "pleasure" to unworldly ones; and Ka limits Akasa to the non-physical.** So thus by making each word would qualify other.

This can be put in the mathematical form as follows:

$$\int_{\text{ख}}^{\text{क}} (\text{Life})\text{Brahman} = \text{Pleasure to unworldly ones}$$

When Life is integrated with respect to Brahman from limits Kha to Ka (Absolute Unity), it becomes Ka.

$$\int_{\text{क}}^{\text{ख}} (\text{Life})\text{Brahman} = \text{Akasa to the non – physical}$$

When Life is integrated with respect to Brahman from limits (Absolute Unity) to kha (Infinity), it becomes Infinity.

Thus, क / Ka (Absolute Unity) = ख / Kha (Infinity) and Ka (Absolute Unity). Kha (Infinity)  
क = ख and ख = क.

Again to shown how the Prana is the greatest to be obtained, is explained (Chapter VII) that –

IPrana (Life) > Aspiration (asha) > Memory (smar) > Ether (akasa) Fire (tej) > Water (ap) > Food (annam) > Strength (balam) > Understanding (Vigyanam) > Contemplation (dhyanam) > Contemplation (dhyanam) > Intelligence (chittam) > Will (sankalpa) > Mind (man) > Speech (vak) > Name (nam).

Here, how the concept of relativity has been applied can be noted in explaining the big, bigger and the biggest of all.

Interestingly, it is explained, immediately after explaining infinitesimal quantities with different examples.

Infinitesimal quantities are also explained in different categories as follows:

- ॐ Honey is collected from different flowers of plants and trees but the taste is the same (6.9.1-2).
- ॐ Ocean contains waters of different rivers, but the colour is the same (6.10.1-2).
- ॐ The water sucked by a tree percolates to all parts for the growth (6.11.1-2).
- ॐ Bunyan tree grows very big. It contains small fruits. If a fruit broken, we get small . If a seed is broken, nothing is seen inside, because it is seed itself (6.12.1-2).

ॐ If salt is mixed with water, it is dissolved and disappeared, but its presence can be understood with its taste (6.13.1-2).

Thus, to understand diminutive / microscopic particle sizes, relative concept is applied for comparison.

**Brahadaranyaka Upanishad** typically brings out the concept of Infinity specifically. It is mentioned that the Almighty manifested in 10 incarnations. Then, many innumerable manifestations followed. **He is tens and thousands and Infinity (dasa cha sahasrani bahun cha anantani)** (2.5.19). Here, not only the "tens "thousands" are related to Infinity, but the word ananta is used to denote Infinity.

Two infinite lines meeting at Infinity is also figuratively brought out. To reach Brahman / Infinity, there are two paths i.e, mind and speech (4.16.1). By walking with leg one or a chariot moving with one wheel, it cannot be reached (4.16.2). Therefore, intact with both legs man should walk and with two wheels chariot should run to reach (the Infinity). That the two wheels of chariot always tread the path of a parallel line is well known fact. Therefore, the over emphasis that "two wheels should run intact to reach" given, is to point out that they (the lines) meet at Infinity.

The following conversation specifically discusses about the nature of Infinity: The king says that one should worship God **ananta - "All-pervasive or Infinite"**. Yajnavalkya asks "**What is the nature of Infinity?**". He replies, "The endless quarters..... to whatever direction one may go, one never gets its end.....it is infinity" (4.1.5).

Another sloka (4.3.22) typically explains that the soul losses its individuality, as in total dissolution with the Supreme Divinity. This implies that though there are many numbers, all will lose their identity in Infinity.

Again Infinity is succinctly explained (4.3.33) by comparing Happiness (ananda) of man with that of Manes, Gandhrva etc, Mathematically follows:

$$\begin{aligned} \text{Human Happiness} &= \frac{\text{Mane Happiness}}{100} \\ &= \frac{\text{Gandnarva's Happiness}}{10,000} \\ &= \frac{\text{God's Happiness (bv birth)}}{10,00,000} \end{aligned}$$

$$\frac{\text{Prajapati-Brahma's Happiness}}{10,00,00,000}$$

$$\frac{\text{Brahma's Happiness}}{10,00,00,00,00,000}$$

Therefore, Brahma's Happiness = Human's Happiness  $\times 10^{12}$ .

When it is expressed in reverse, we can write that Human happiness is  $1 \times 10^{-10}$  of that of Brahman. Similarly, the human years with of Brahman are compared:

- ⊙ 1 Brahman day timer = 4.32,00,00,000 human years
- ⊙ 1 Brahman night time = 4,32,00,00,000 human years
- ⊙ Therefore, 1 Brahman day = 8.64.00.00,000 human years
- ⊙ From this, the Brahma's age =  $1.5766 \times 10^{14}$  human years
- ⊙ Or one human year =  $0.6342 \times 10^{-14}$

Here the important point is how the infinites and infinitesimals are compared and explained figuratively. Thus, ananda (happiness) becomes ananta (Infinity). Though ananda is immeasurable, it is quantified for comparison. Then, Brahma's happiness is quantified in comparison to man, though Brahma cannot have any Happiness. In fact, anandananta (Infinite Bliss) is mentioned in the context of divinity. Therefore, by giving such quantification, Infinity is clearly implied. Similarly, the same equations can be put in the reverse, saying that Man's happiness is 100 times that of Man and so on. This proves the ability of scriptures conveying the concept of relativity.

Brahma Sutras use the word "bnúma" to denote Infinity and it is equated with Brahman (1.3.8). Individual and Soul becomes one with the Infinite (3.2.26). Though Jiva and Brahman appear to be different, they are connected together. It is explained in this way that "....." on account of both (i.e, the difference and non-difference being taught by the Sruti), the relation of Jiva and Brahman is to be taken like that between a serpent and its coils" (3.2.27).

First, Jiva (Zero) is equated with Brahman (the Absolute) and then, it is said that they are equal in Infinity ( $0 + 1 = 1$ ) and  $0 + \infty = \infty$ ,  $1 + \infty = \infty$ ). Then, the relation between Jiva and Brahman compared with the Serpent (head resembles Zero) and its coils (resembles Infinity). Here perhaps the relation between Zero and Infinity is brought out clearly and incidentally, the symbol of Infinity is the coil of a serpent mentioned can also be noted.

Here, it is pertinent to quote Carl Sagan (20) :

"The Hindu religion is the only one of the World great faiths dedicated to the idea that the Cosmos itself undergoes an immense indeed an infinite, number of deaths and rebirths. It is the only religion in which the time scales correspond. no doubt by accident, to those of modern scientific cosmology. Its cycles run from our ordinary day and night to a day and night of Brahma, 8.64 billion years long, longer than the age of Earth or the Sun and about half the time since the Big Bang. and there are much longer time scales still.

"There is the deep and appealing notion that the universe is but the dream of the god who, after a hundred Brahma years, dissolves himself into a dreamless sleep. The Universe dissolves with him - until Brahma century, he stirs, recomposes himself and again begin to dream the great Cosmic dream. **Meanwhile there are an infinite number of other universes, each with its own god dreaming the cosmic dream.** These great ideas are tempered by another, perhaps still greater. It is said that men may not be the dreams of the gods but rather that the gods are the dreams of men".

The time scale was not at all accidentally conceived, as has been perceived by him, but the thought processes worked behind such concept revealed through the available Scriptures vouchsafe such evolution of concept.

**Infinity and Limits:** In mathematics (series and integration), to define or analysis the nature of a number, series or expansion, limits are applied, when their values are increased infinitely great or infinitesimally small.

Infinity is a quantity whose limit is equal to "Immeasurable", because it is a variable and its absolute value increases without bound, but an Infinitesimal is a quantity whose quantity may approach zero, but not equal to zero. In fact, it is also defined as the number greater than zero, but lesser than  $1/2$ ,  $1/3$ ,  $1/4$ ...

If  $x$  is an infinitely large quantity. then  $1/x$  is an infinitely Small quantity (infinitesimal); if  $x$  is an infinitesimal, then,  $1/X$  is an infinitely large quantity.

The number  $n$  is called the limit of the sequence  $n_1, n_2., n_3, n_4 \dots$  if  $n_N$  approaches  $N$  without bound, as the number  $n$ , then  $n_3$ , increases.

Limit  $n_N = n$   
or expanded. Limit  $n_N = n, n \rightarrow \infty$

The number  $n$  is called the limit of the function  $f(x)$ , as  $x \rightarrow 1$  (i.e.  $x$  approaches or tends to 1), as  $x$  approaches from the right or the left, the value of  $f(x)$  approach to)  $n$  without bound.

Limit  $f(x) = n, X \rightarrow l$

From Hiranaya garba, everything is created and during pralaya, all vanish into waters. Again from waters, the creation starts with Hiranyagarba. This leads to the proposition that all numbers originate from zero and end merges with Infinity to become Infinity. Infinity merges with infinity to become infinity to continue the cycle of creation and destruction or birth and death.

**The Relation of Infinity with Exact Sciences:** the infinity played crucial role in Mathematics and Physics with discoveries and inventions. The entire modern physics deals Infinity.

1. **When mass becomes infinity:** In the variation of mass with velocity, the following equation used:

$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$	<p>Where, m = effective mass  M (0) = rest mass  V = velocity of the body whose mass is m  c = velocity of light</p>
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Here, if  $v = c$ , i.e, if the body attains the velocity of light, then, m becomes  $\infty$ .

2. **Renormalization:** According to Heisenberg uncertainty principle, the law of conservation of energy is not strictly observed for short periods of time. Therefore, an electron may briefly emit and then reabsorb other particles like photons. These particles are transient in nature and called virtual particles and the mass increased due to them is called self mass. Unfortunately, when physicists in the 1930s tried to calculate the Self mass due to virtual photons, they got infinite results. Hendrik Kramers suggested that infinite self mass might combine with an infinite intrinsic to give the finite observed mass. Then, the observable quantities are expressed in terms of their sum, avoiding the problem of infiniteness. Such an avoidance of infiniteness is known as "Renormalization".

3. **Big bang theory and matter with infinite density:** The density of matter was considered infinite, when the universe was created according to the Big band theory applying relativity to cosmology.

4. **The size of the Universe – Infinite:** In determination of the size of the Universe, the scientists play with zeroes. They claim the diameter of the observable Universe is 25 million light years = 250,000,000,000,000,000 kms (21)! It can very well be said approaches Infinity.

5. The geological scales are also in millions of years.

6. Two parallel lines are said to be meeting at infinity in geometry of two or more dimensions. In analytical geometry, curve may touch a line called asymptote as a variable approaches infinity.

7. Georg Cantor (1845-1918) invented set theory and discovered the transinfinite numbers related to the concept of infinity. In set theory, a finite set is a set that has no proper subset equivalent to itself and an infinite set has at least one proper subset equivalent to itself.

8. About the Ether, Cosmic Rays and connected topics much philosophy is applied than science or philosophical principles are applied in the scientific garb. They are always thought to be originated from the Universe at Infinite distance. This mystification of the science has thus made man to realize the power of nature over man. And that is why perhaps, Einstein rightly marked philosophy starts where Science ends.

**Infinity in Astronomy and Geography:** Of the nine planets (Navagrahas), Rahu and Ketu are nothing but parts of snake. Ketu has the head portion and Ragu the body portion. They actually represent the ascending and descending nodes of Moon.

In the churning of ocean, the Maha Meru was used as stick. In fact, Ananta lifted the Meru! The Maha Meru is a perfect three dimensional figure containing all forms of Geometry (sphere, trepzoids, cubes etc). It is nothing but elevation of Sri Chakra drawn in two dimensions dot (containing dot, line, curves, circles, hyperbolas etc). Therefore, the association of Meru with Ananta, particularly, churning is interesting, implying Infinity controls all forms of Geometry in two and three dimensions. The churning has mathematical significance, with all geometrical figures was rotated right to left and left to right, each figure appears differently. The entire figure would appear as Solid Cone. Triangles and polygons appear as circles. Therefore, It is a clear indication that if the sides of polygon is increased to Infinity, it becomes a circle! Note that Meru is rotated by Ananta the Infinity. If the radius of a that Circle reduced smaller and smaller, it becomes a dot / point which is nothing but the bindu at the top of Meru and centre of Sri Chakra.

Again, the Meru is the Centre point of the World for geographical and astronomical purpose. The Ananta - Infinity protects the entire world.

**Infinity in Art and Architecture:** In understanding the mystery Of God, creation of Universe. Origin of Man etc., Indian seers have conducted many experiments leading to composing many compendiums in all subjects. Later, they were represented in the figure forms for the understanding of common people. Those who know about Infinity, Zero, Numbers and their operations need not have pen and paper to write them to discuss or understand operations. But, for students or those who know about them the first time, all have to be represented in figure forms.

Thus, Sculptors, painters and other artists started representing the philosophical concepts in art form. In the Indian art, the representation of infinity is visualized with unity, blue color, sky, lotus, pointed conical and trapezoidal figures directed towards sky. In fact, all temples, Buddha Viharas, bell tops and puja utensils. Maha Meru, heads of Gods, particularly kiritas pointed ends. At the bottom with so many paraphernalia, the structure terminates to a tip pointing towards sky (take the example of two dimensional representation of Sri Chakra and the dimensional Maha Meru). Lotus is symbolically Infinity with increasing petals. It is profusely used in all art forms and also in yogic mandalas.

**Modern Scientists and Infinity:** Modern science has enormously benefitted, enriched and refined with the concept and application of infinity. That Modern physicists and mathematicians adapted and adopted it exhaustively to postulate hypotheses, develop theories, formulate laws and discover / Invent many devices all of sudden within 50 years is definitely questionable or debatable in the state of Finiteness.

For the intellectual with new thinking processes, even a small hint or spark will lead to invention and discovery. Therefore, it may not be exaggerated to mention that they were thrilled to read the ancient Scriptures of Indians to be immediately and applied in their works. Albert Einstein, De Broglie, Oppenheimer and other scientists openly have acknowledged indebtedness to the Indian scriptures.

Historians and economists have now proved beyond doubt that with the so called Industrial Revolution of England coincides with the looting of Bengal. Similarly, the same period coincided with carrying away of many mathematical and astronomical tables, charts and diagrams by the Europeans and to their respective countries. Therefore, definitely, the scientists must have been activated by them to quicken the Scientific Revolution in the half of 20<sup>th</sup> century.

**Philosophical Infinity led to the Invention Infinity:** Indian seers recognized the following aspects of Universe, World and matter in time and space:

1. The existence of the world (anda) in space (Brahmanda), its creation from akasa, infinite variety of space structures of matter and the essential non-exclusiveness of all material systems (different worlds above and below the world), creation of man, flora and fauna etc have been the uniqueness of the cosmology. In fact, the Puranas typically mention the existence of many worlds in the universe.
2. The existence of the world in time (in terms of yugas time scale of Brahma), the conservation of matter and energy, (as one form always transforms into other form), the eternity of its existence (Shruti and Pralaya connected with the destruction and creation with life of cyclic nature) are also vividly explained.

3. The quantitative inexhaustibility of matter in depth, the infinite variety of its qualities, interrelations, forms of existence and tendencies of development (such figurative representation led to the different Gods and Goddesses) are recognized.
4. The qualitative heterogeneity of the structure of matter, the existence of innumerable qualitatively different levels of the structural organisation of matter, which possesses at each level different specific properties and its subject to such different laws (man's approach to such different forms again leads to convergence) are also appreciated.
5. All innumerable living and non-living things are treated equally and for welfare of all, the entire universe prayer is directed towards the Infinity. The theoretical understanding of Infinity develops with the progress of scientific knowledge. Particularly, the measurement of time has been very unique Indian system. It has been associated and calculated with following:
  - a. With the activities of man
  - b. With the activities of Nature and
  - c. With the activities of Universe

Thus, in his attempt to comprehend big, bigger and biggest numbers lead to the invention of Infinity.

**Conclusion:** The mathematical implication of operations of zero and infinity have been clearly dealt with by the ancient Indian mathematicians for the first time in history. The conceptual evolution and development is traced back to Vedic (3500 BCE and Upanishadic (2000 BCE) periods. The first mathematical attempt to divide a number by zero to get Infinity was made by Brahmagupta (628 CE). Bhaskara II (1150 CE) was the first to use Infinity the mathematical operations. The basis for all such operations supported the scriptural evidences explained. The liberal, progressive and universal thinking of the Indians made them to finite infinities and infinite finites leading to meaningful mathematical dialogue forever in all modern scientific thinking processes.

**Suggestion:** It is unfortunate that western bias against India has been continuing in Encyclopedias, reference works and standard books, in spite of the spread and availability of information and data about the sciences and arts of India. The western writers have tendency to glorify Greeks for every scientific thinking and adamantly or otherwise refuse to acknowledge the advancement of Indians made, just because they conceived such ideas before the Greeks. Another dangerous trend noted is that they tactful enough to expunge, expunge and edit all Indian Contributions to the development of science and technology in the history of science. Here, the Indians should be careful to counter their moves. The resourceful Indians can do the needful in publishing and creating websites for all ancient works on Science and Technology with commentaries written by scholars, so that a correct perspective presented to the world. Therefore, the ancient texts are scrutinized with modern scientific thought, we may get new results leading to new inventions and discoveries can be made for the benefit of humankind.

## Notes and References

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2. Ted Honderich (Ed.), **The Oxford Companion to Philosophy**, 1995, USA; p.408.
3. Sudhakara Dvivedi (Ed.), **"Bijaganita of Bhaskara II"**, Benares, 1927, pp.5-6.
4. Bibhutibhusan Datta and Avadesh Narayan Singh, **"History Of Hindu Mathematics"**, Asia Publishing House. Bombay, 1962, p.244.
5. Ibid. p.244-245.
6. V. S. Apte, **Sanskrit-English Dictionary**, Motilal Banarasi-, New Delhi. 1963.
7. B. Datta and A.N. Singn, opt.cit. Translation of Colebrooke Given.
8. The Fallaci of Algebraic operations:

$x = y.$ Then $x^2 = xy.$ Subtract the same thing from both sides: $x^2 - y^2 = xy - y^2.$ Dividing by $(x-y)$ , obtain $x + y = y.$ Since $x = y$ , we see that $2y = y.$ Thus $2 = 1$ , since we started with $y$ nonzero. Subtracting 1 from both sides, $1 = 0.$	$b = a$ $ab = a^2$ $ab - b^2 = a^2 - b^2$ $b(a - b) = (a + b)(a - b)$ $b = a + b$ $b = b + b$ $b = 2b$ $1 = 2$
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But, it is absurd, because, two variables are equated on imaginary ground and the algebraic operations are carried on..

9. Florian Cajori, **A History of Mathematical Notations**, The Open Court Publishing Co., London, 1923 p.214
10. Karl Menninger. **"Number words and Number Symbols - A Cultural History of Numbers"**. The MIT Press. 1969. U.K, D.245).
11. Bernard Fontenelle in the early 18<sup>th</sup> century preferred a more categorical position: in as much as there are infinitely many natural numbers, an infinite number exists as truly as do finite numbers; and the reciprocal of infinity is an infinitesimal.

George de Buffon (1707-1788) however rejected his infinity as an illusion, seeing it in the infinitely large and small nothing but the idea of a quantity increasing or decreasing without limit.

Jean d Alembert (1717-1783), writing the article "Differential for the Encyclopedie, decried the existence of an infinity except in the negative sense of a limit of finite quantities (Colliers Encyclopedia. Vol.13, 1982, p.5-6).

12. Printed from the originals in the collection of the Right Honourable the Earl of Macclesfield "**Correspondence of Scientific Men" of the seventeenth century including letters of Barrow. Flamsteed, Wallis and Newton**, Oxford, At the University Press, M.DCCC.XL1.

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15. Florian Cajori, "**A History of Mathematics**", The Macmillan Co.. USA. 1924, p.257.

16. C.F. Gauss, "**Brief on Schumacher, Werke**". Bd, 8, 2161; quoted from Moritz, Memorabilia mathematica, 1914, p.337  
Also Florian Cajori, opt.cit. p.446.

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18.Cantor. **Zum Problem actualen Unendlichen. Natur and Offenbarung** ", Bd.32, 1886, p.226; quoted from Moritz, Memorabilia mathematica, 1914, p.337. also Florian Cajori, opt.cit, p.447).

17. All references from Upanishads are taken from the publications of Sri Ramakrishna Mutt, Gaudiya Mutt, Sri Chinmaya Mission etc.

20. Carl Sagan. "**Cosmos**", Macdonald & Co Ltd.. 1983. U.K, p.258.

21. 1 AU = 149,600.000 km

1 Light year = 60.000 AU

= 60.000 x 149.600,000

= 89.760.000.000.000

= 90 billion (approximately) Kms

Therefore, 25 million Light years

= 25.000.000 x 90,000,000,000,000

= 2,250,000,000,000,000,000